$$=\tfrac{2}{3}\frac{\phi}{\psi}\left[1+\frac{\psi^2}{6}-\frac{\phi^2}{6}+\frac{\psi^4}{36}-\frac{\psi^4}{120}+\frac{\phi^4}{120}-\frac{\phi^2\psi^2}{36}\right]\cdots$$

So, $$\tfrac{1}{3}\frac{\tan\phi}{\tan\psi}=\tfrac{1}{3}\frac{\phi}{\psi}\left[1+\left(\frac{\phi^2}{3}+\frac{2\phi^4}{15}\right)\right]\left[1+\left(\frac{\psi^2}{3}+\frac{2\psi^4}{15}\right)\right]^{-1}\cdots$$

$$=\tfrac{1}{3}\frac{\phi}{\psi}\left[1+\frac{\phi^2}{3}-\frac{\psi^2}{3}-\frac{2\psi^2}{15}+\frac{\psi^4}{9}+\frac{2\phi^4}{15}-\frac{\phi^2\psi^2}{9}\right]\cdots$$

Their sum $$=\frac{\phi}{\psi}\left[1+\frac{\psi^4}{180}-\frac{10\phi^2\psi^2}{180}+\frac{9\phi^4}{180}\right]\cdots$$

$$=\frac{\phi}{\psi}+\frac{\phi}{\psi}\cdot\tfrac{1}{180}[\phi^2-\psi^2][9\phi^2-\psi^2].$$

In Mr. Greenwood's solution it will be noticed that $\dfrac{\tan\phi}{\tan\psi}$ is *not* AB as stated.

The relation $\tan\phi=\phi+\frac{1}{3}\phi^3+\frac{2}{15}\phi^5+\frac{17}{315}\phi^7\ldots$ is readily obtained from

$$\tan\phi=\left[\phi-\frac{\phi^3}{6}+\frac{\phi^5}{120}-\frac{\phi^7}{5040}+\ldots.\right]\left[1-\frac{\phi^2}{2}+\frac{\phi^4}{24}-\frac{\phi^6}{720}+\ldots.\right]^{-1}.$$

The second factor $$=\left[1+\left(\frac{\phi^2}{2}-\frac{\phi^4}{24}+\frac{\phi^6}{720}\right)+\left(\frac{\phi^2}{2}-\frac{\phi^4}{24}\right)^2+\left(\frac{\phi^2}{2}\right)^3+\ldots.\right]$$

and on multiplying out the expansion is obtained to any power we require.

PROBLEMS FOR SOLUTION.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

157. Proposed by A. H. HOLMES, Brunswick, Maine.

Find integral values for m and n in $64m^2n^2(m^2-n^2)^2+(m^2+n^2)^4=\square$.

158. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

Find positive rational values of a and b in the equation $x^4-2ax^2+x+a^2-b=0$, that will make each of the roots (all different) rational numbers.